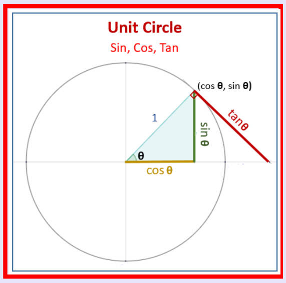
Math 241

Winter 2023

Lecture 2



Class QZ 1:

Solve $\boxed{3x^2 - 4x - 7 = 0}$ by Quadratic Formula.

Solution Set

$a = 3$, $b = -4$, $c = -7$

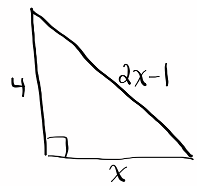
$b^2 - 4ac = (-4)^2 - 4(3)(-7) = 16 + 84 = 100$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{100}}{2(3)} = \frac{4 \pm 10}{6}$$

$x = \frac{4+10}{6} = \frac{14}{6} = \boxed{\frac{7}{3}}$ ✓ $\quad x = \frac{4-10}{6} = \frac{-6}{6} = \boxed{-1}$ ✓

$$\boxed{\left\{-1, \frac{7}{3}\right\}}$$

Find x:



Right Triangle

Use $a^2+b^2=c^2$

$4^2+x^2=(2x-1)^2$

$16+x^2=(2x)^2-2(2x)(1)+1^2$

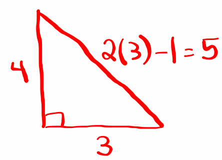
$16+x^2=4x^2-4x+1$

Recall:
$(A-B)^2=A^2-2AB+B^2$

$4x^2-4x+1-16-x^2=0$

$3x^2-4x-15=0$

$(3x+5)(x-3)=0$

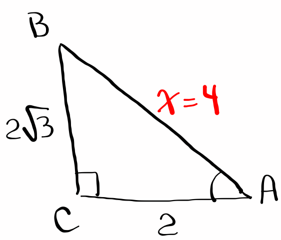


By Zero-Product rule

$3x+5=0$ ⋮ $\cancel{x=-5/3}$

$x-3=0$ ⋮ $x=3$

Find the missing side, then complete the chart below



$\sin A=\frac{\sqrt{3}}{2}$	$\csc A=\frac{2\sqrt{3}}{3}$
$\cos A=\frac{1}{2}$	$\sec A=2$
$\tan A=\sqrt{3}$	$\cot A=\frac{\sqrt{3}}{3}$

$(2\sqrt{3})^2+2^2=x^2$

$4\cdot 3+4=x^2$

$x^2=16$ $\boxed{x=4}$

$\sin A=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$

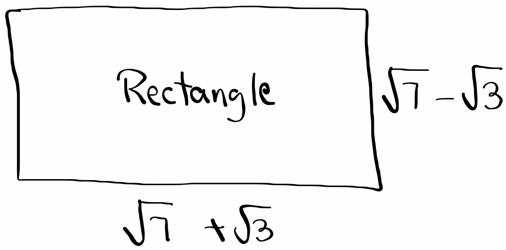
$\csc A=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\cos A=\frac{2}{4}=\frac{1}{2}$ $\sec A=\frac{2}{1}=2$

$\tan A=\frac{2\sqrt{3}}{2}=\sqrt{3}$

$\cot A=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{9}}=\frac{\sqrt{3}}{3}$

Find area & Perimeter:



$A = LW$

$P = 2L + 2W$

$$A = \underbrace{(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})}_{\text{Conjugates}}$$

$$= (\sqrt{7})^2 - (\sqrt{3})^2$$

$$= 7 - 3 = \boxed{4}$$

Recall:

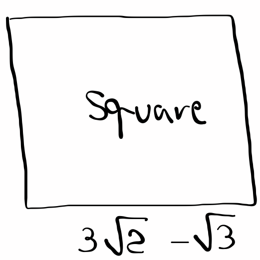
$$(A + B)(A - B) = A^2 - B^2$$

$$P = 2L + 2W = 2(\sqrt{7} + \sqrt{3}) + 2(\sqrt{7} - \sqrt{3})$$

$$= 2\sqrt{7} + \cancel{2\sqrt{3}} + 2\sqrt{7} \cancel{- 2\sqrt{3}} = \boxed{4\sqrt{7}}$$

Find Area & Perimeter:

$A = S^2$, $P = 4S$



$$A = (3\sqrt{2} - \sqrt{3})^2$$

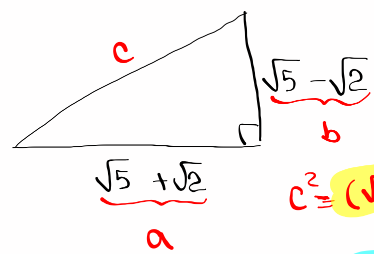
$$= (3\sqrt{2})^2 - 2(3\sqrt{2})(\sqrt{3}) + (\sqrt{3})^2$$

$$= 9 \cdot 2 - 6\sqrt{6} + 3$$

$$= 18 - 6\sqrt{6} + 3 = \boxed{21 - 6\sqrt{6}}$$

$$P = 4S = 4(3\sqrt{2} - \sqrt{3}) = \boxed{12\sqrt{2} - 4\sqrt{3}}$$

Find the hypotenuse:



$c^2 = a^2 + b^2$

$= (\sqrt{5}+\sqrt{2})^2 + (\sqrt{5}-\sqrt{2})^2$

$c^2 = (\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{2}) + (\sqrt{2})^2 +$

$(\sqrt{5})^2 - 2(\sqrt{5})(\sqrt{2}) + (\sqrt{2})^2$

$c^2 = 5 + \cancel{2\sqrt{10}} + 2 + 5 - \cancel{2\sqrt{10}} + 2 = 14$

$c^2 = 14 \rightarrow \boxed{c = \sqrt{14}}$

Please Review

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)(A-B) = A^2 - B^2$

Simplify:

$$(\overbrace{\sin x \csc x}^{1} + \overbrace{\cos x \sec x}^{1})^2 - 4\overbrace{\tan x \cot x}^{1}$$

Hint: Recognize reciprocal functions

$\csc x = \frac{1}{\sin x} \Rightarrow$ Cross-Multiply $\sin x \csc x = 1$

$\sec x = \frac{1}{\cos x} \Rightarrow$ Cross-Multiply $\cos x \sec x = 1$

$\cot x = \frac{1}{\tan x} \Rightarrow \tan x \cdot \cot x = 1$

$(\sin x \csc x + \cos x \sec x)^2 - 4 \tan x \cot x$

$= (1 + 1)^2 - 4 \cdot 1 = 2^2 - 4 = 4 - 4 = \boxed{0}$

Do not use Ø for 0.

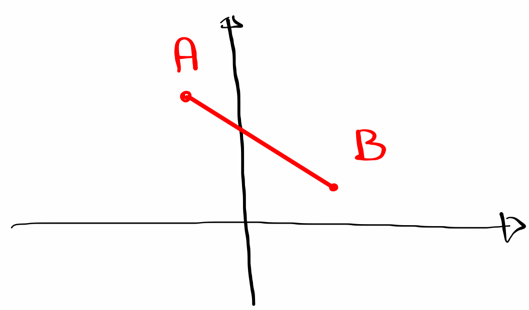
Simplify:

$$\boxed{(\sin A + \cos A)^2} + \boxed{(\sin A - \cos A)^2}$$

$$= \sin^2 A + \cancel{2\sin A\cos A} + \cos^2 A +$$
$$\sin^2 A - \cancel{2\sin A\cos A} + \cos^2 A = 1 + 1$$

$$= \boxed{2}$$

Plot $A(-1,5)$ and $B(2,1)$, then find the distance between them.

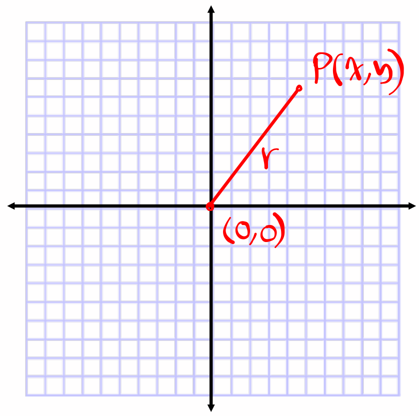


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1-2)^2 + (5-1)^2}$$

$$= \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$$

Plot the Point $P(x,y)$, then find its distance from the origin.



$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = \sqrt{(x-0)^2 + (y-0)^2}$

$\boxed{r = \sqrt{x^2 + y^2}}$

Square both Sides

$r^2 = \left(\sqrt{x^2 + y^2}\right)^2$

$r^2 = x^2 + y^2$

$x^2 + y^2 = r^2$

Circle

center $(0,0)$, radius r

Draw $(x+3)^2 + (y-4)^2 = 25$

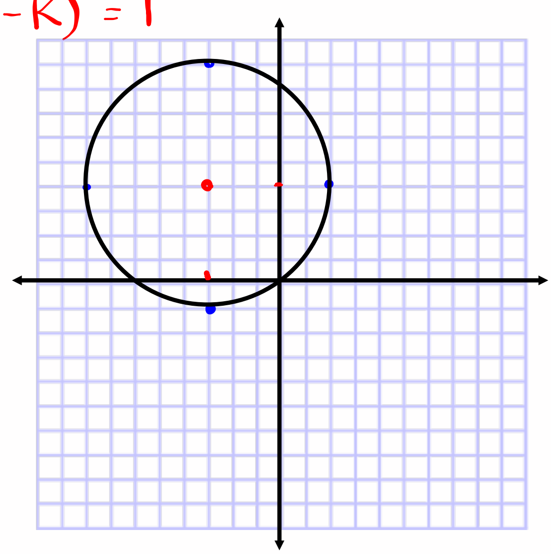
$(x-h)^2 + (y-k)^2 = r^2$

$h = -3$

$k = 4$

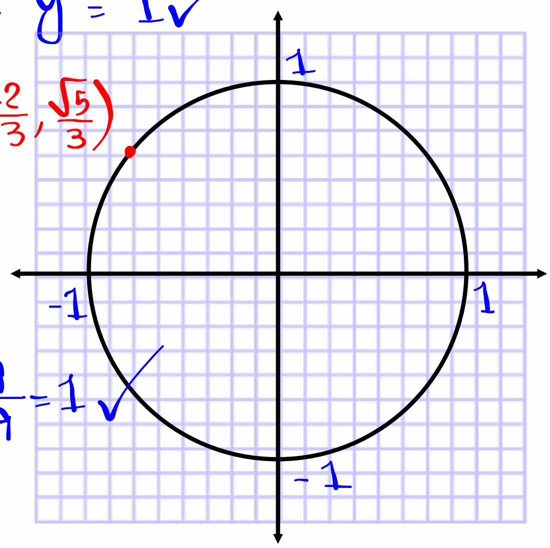
$\Rightarrow$ Center $(-3,4)$

$r^2 = 25 \Rightarrow r = 5$



Verify that the point $\left(\frac{-2}{3}, \frac{\sqrt{5}}{3}\right)$ is on the Unit Circle. $\to$ QII

Center $(0,0)$
Radius 1 $\Rightarrow x^2 + y^2 = 1$ ✓



Plug in the Values

$$\left(\frac{-2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} + \frac{5}{9}$$

$$= \frac{4+5}{9} = \frac{9}{9} = 1 \checkmark$$

Verify: $\cos x \cdot \tan x = \sin x$ ✓

Recall $\tan x = \frac{\sin x}{\cos x}$

$$\cos x \cdot \tan x = \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} = \sin x \checkmark$$

Verify $\cos x \cdot \csc x \cdot \tan x = 1$ ✓

$$\cancel{\cos x} \cdot \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} = \boxed{1}$$

Hint:

$\csc x = \frac{1}{\sin x}$, $\tan x = \frac{\sin x}{\cos x}$

Verify $\quad \csc A \tan A = \sec A \checkmark$

$$\frac{1}{\cancel{\sin A}} \cdot \frac{\cancel{\sin A}}{\cos A} = \frac{1}{\cos A} = \sec A \checkmark$$

$\sin^2 A + \cos^2 A = 1 \qquad \csc A = \dfrac{1}{\sin A}$

$1 + \tan^2 A = \sec^2 A \qquad \sec A = \dfrac{1}{\cos A}$

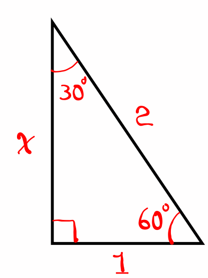
$1 + \cot^2 A = \csc^2 A$

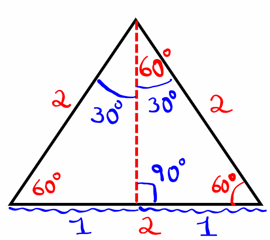
$\tan A = \dfrac{\sin A}{\cos A}, \quad \cot A = \dfrac{\cos A}{\sin A} \qquad \cot A = \dfrac{1}{\tan A}$

Special Right Triangles:

1) $30° - 60° - 90°$

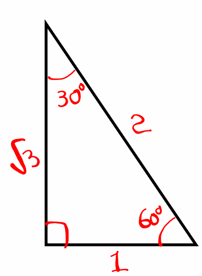
2) $45° - 45° - 90°$



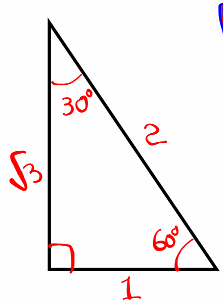


$x^2 + 1^2 = 2^2$

$x^2 = 3 \rightarrow x = \sqrt{3}$



$$\boxed{\begin{array}{ll} \sin 30° = \frac{1}{2} & \csc 30° = 2 \\ \cos 30° = \frac{\sqrt{3}}{2} & \sec 30° = \frac{2\sqrt{3}}{3} \\ \tan 30° = \frac{\sqrt{3}}{3} & \cot 30° = \sqrt{3} \end{array}}$$

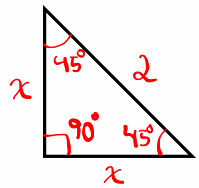


$\sin 60° = \frac{\sqrt{3}}{2}$ $\csc 60° = \frac{2\sqrt{3}}{3}$

$\cos 60° = \frac{1}{2}$ $\sec 60° = 2$

$\tan 60° = \sqrt{3}$ $\cot 60° = \frac{\sqrt{3}}{3}$

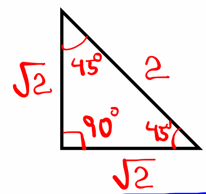
45°–45°–90°



$x^2 + x^2 = 2^2$

$2x^2 = 4$

$x^2 = 2 \rightarrow x = \sqrt{2}$



$\sin 45° = \frac{\sqrt{2}}{2}$ $\csc 45° = \sqrt{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$ $\sec 45° = \sqrt{2}$

$\tan 45° = 1$ $\cot 45° = 1$

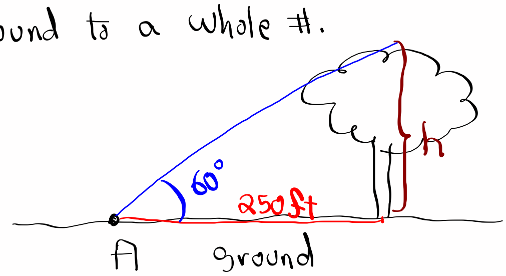
	30°	45°	60°
Sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

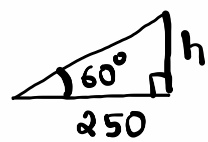
Point A is 250 ft from a tall tree.

Point A is on the ground.

At point A, angle between ground and top of the tree is 60°. How tall is the tree.

Round to a whole #.





$\tan 60° = \dfrac{h}{250}$

Cross - Multiply $\quad h = 250 \cdot \tan 60°$

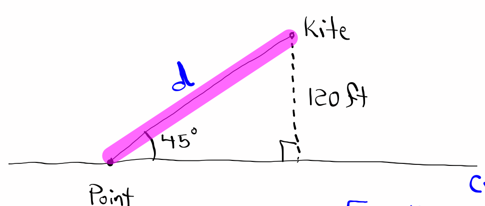
$= 250 \cdot \sqrt{3} = 433.0127...$

About 433 ft

A kite is flying at 120 ft high.

From a point on the ground, angle of elevation to the kite is 45°. → When you look up

How far is the kite from that point on the ground? Round to 1-decimal.



$\sin 45° = \dfrac{120}{d}$

$\dfrac{\sqrt{2}}{2} = \dfrac{120}{d}$

Cross - Multiply

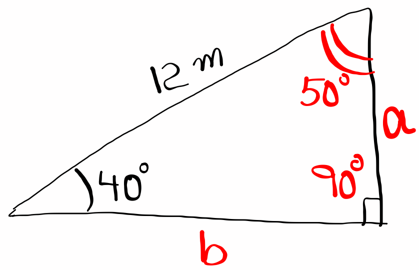
$\sqrt{2}\, d = 240$

$d = \dfrac{240}{\sqrt{2}}$

$d = 169.7056...$

$d \approx \boxed{169.7 \text{ ft}}$

Solve the triangle below: → Find missing Sides and angles.



$\sin 40^\circ = \frac{a}{12}$

$a = 12 \cdot \sin 40^\circ$

$a = 7.7134\ldots$

$\boxed{a \approx 8}$ m

$\cos 40^\circ = \frac{b}{12}$

$b = 12 \cdot \cos 40^\circ$

$b = 9.1925\ldots$

$\boxed{b \approx 9}$ m

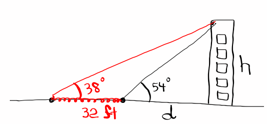
My angle of elevation to the top of a building was 54°.

I walked away from the building 32 ft, and my angle of elevation became 38° to the top of the building.

How far was I from the building before walking away from the building?

$\tan 54^\circ = \frac{h}{d}$

$h = d \tan 54^\circ$



$\tan 38^\circ = \frac{h}{d+32}$

$h = (d+32) \cdot \tan 38^\circ$

Find d.

$(d+32)\tan 38^\circ = d \tan 54^\circ$

$d \tan 38^\circ + 32 \tan 38^\circ = d \tan 54^\circ$

$32 \tan 38^\circ = d \tan 54^\circ - d \tan 38^\circ$

$32 \tan 38^\circ = d(\tan 54^\circ - \tan 38^\circ)$

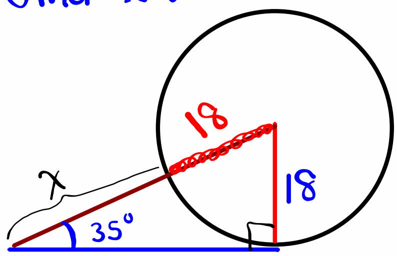
$\frac{32 \tan 38^\circ}{\tan 54^\circ - \tan 38^\circ} = d$

$d = 42.01192362$

$\boxed{d \approx 42 \text{ ft}}$

Please Make Sure to Know how to use Your Calc.

Find x:



$$\sin 35^\circ = \frac{18}{x+18}$$

$$(x+18)\cdot \sin 35^\circ = 18$$

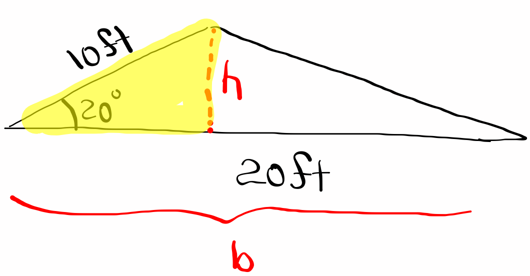
$$x \sin 35^\circ + 18\cdot \sin 35^\circ = 18$$

$$x \sin 35^\circ = 18 - 18\sin 35^\circ$$

$$x = \frac{18 - 18\sin 35^\circ}{\sin 35^\circ}$$

$$x = 13.38204 \qquad \boxed{x \approx 13}$$

Find the area of the triangle below:



$$A = \frac{bh}{2}$$

$$= \frac{20\cdot h}{2}$$

$$\sin 20^\circ = \frac{h}{10} \Rightarrow h = 10\sin 20^\circ$$

$$A = \frac{\cancel{20}^{\,10}\cdot 10\cdot \sin 20^\circ}{\cancel{2}}$$

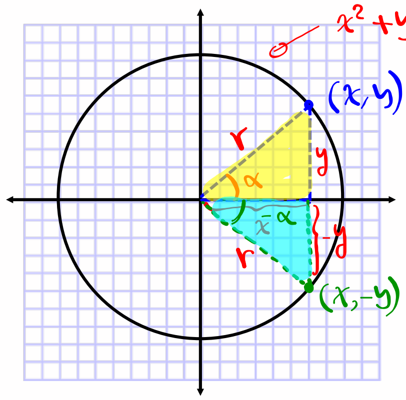
$$= 100\cdot \sin 20^\circ$$

$$= 34.20201433$$

$$\boxed{\text{Area} \approx 34\ \text{ft}^2}$$

Consider point (x, y) in QI, and it is on the Circle center at $(0,0)$ with radius r.

$x^2+y^2=r^2$



Alpha α

$\sin\alpha = \dfrac{y}{r} \Rightarrow \tan\alpha = \dfrac{y}{x}$

$\cos\alpha = \dfrac{x}{r}$

$\sin(-\alpha) = \dfrac{-y}{r} = -\sin\alpha$

$\cos(-\alpha) = \dfrac{x}{r} = \cos\alpha$

$\tan(-\alpha) = \dfrac{-y}{x} = -\tan\alpha$

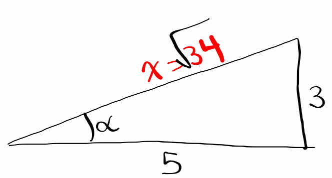
we showed

$$\boxed{\begin{aligned} \sin(-\alpha) &= -\sin\alpha \\ \cos(-\alpha) &= \cos\alpha \\ \tan(-\alpha) &= -\tan\alpha \end{aligned}}$$

$\sin(-30^\circ) = -\sin 30^\circ = -\dfrac{1}{2}$

$\cos(-60^\circ) = \cos 60^\circ = \dfrac{1}{2}$

$\tan(-45^\circ) = -\tan 45^\circ = -1$



Missing Side

$x^2 = 3^2 + 5^2 = 9 + 25 = 34$

$x = \sqrt{34}$

$\sin(-\alpha) = -\sin\alpha = -\dfrac{3}{\sqrt{34}} = \boxed{-\dfrac{3\sqrt{34}}{34}}$

$\cos(-\alpha) = \cos\alpha = \dfrac{5}{\sqrt{34}} = \boxed{\dfrac{5\sqrt{34}}{34}}$

$\tan(-\alpha) = -\tan\alpha = -\dfrac{3}{5}$

Simplify:

$$\sin^2(-\alpha) + \cos^2\alpha =$$

$$(-\sin\alpha)^2 + \cos^2\alpha =$$

$$\sin^2\alpha + \cos^2\alpha = \boxed{1}$$

Simplify:

$$\cos\alpha \cdot \sec(-\alpha) + \tan(-\alpha)\cdot\cot\alpha =$$

$$\cos\alpha \cdot \frac{1}{\cos(-\alpha)} + -\tan\alpha \cdot \cot\alpha =$$

$$\cancel{\cos\alpha} \cdot \frac{1}{\cancel{\cos\alpha}} - \cancel{\tan\alpha} \cdot \frac{1}{\cancel{\tan\alpha}} =$$

$$1 - 1 = \boxed{0}$$

Verify: $\sec\alpha - \cos(-\alpha) = \dfrac{\sin^2\alpha}{\cos\alpha}$ ✓

$$\sec\alpha - \cos\alpha =$$

$$\frac{1}{\cos\alpha} - \cos\alpha =$$

$$\frac{1}{\cos\alpha} - \frac{\cos\alpha \cdot \cos\alpha}{\cos\alpha} =$$

$$\frac{1 - \cos^2\alpha}{\cos\alpha} = \frac{\sin^2\alpha + \cancel{\cos^2\alpha} - \cancel{\cos^2\alpha}}{\cos\alpha}$$

$\sin^2\alpha + \cos^2\alpha$

$$= \frac{\sin^2\alpha}{\cos\alpha}$$ ✓

Verify $\cos\alpha \cot\alpha + \sin\alpha = \csc\alpha$ ✓

$\cos\alpha \cot\alpha + \sin\alpha =$

$$\cos\alpha \cdot \frac{\cos\alpha}{\sin\alpha} + \sin\alpha = \frac{\cos^2\alpha}{\sin\alpha} + \frac{\sin\alpha \cdot \sin\alpha}{\sin\alpha}$$

$$= \frac{\cos^2\alpha}{\sin\alpha} + \frac{\sin^2\alpha}{\sin\alpha}$$

$$= \frac{\cos^2\alpha + \sin^2\alpha}{\sin\alpha} = \frac{1}{\sin\alpha} = \csc\alpha \checkmark$$

Verify $\cos\alpha(\sec\alpha - \cos\alpha) = \sin^2\alpha$ ✓

$$\cos\alpha(\sec\alpha - \cos\alpha) = \cos\alpha \cdot \boxed{\sec\alpha} - \cos^2\alpha$$

$$= \cancel{\cos\alpha} \cdot \frac{1}{\cancel{\cos\alpha}} - \cos^2\alpha$$

$$= \boxed{1} - \cos^2\alpha$$

$$= \sin^2\alpha + \cancel{\cos^2\alpha} - \cancel{\cos^2\alpha}$$

$$= \boxed{\sin^2\alpha} \checkmark$$

Verify $\cot\alpha + \tan\alpha = \csc\alpha \cdot \sec\alpha$

$$\cot\alpha + \tan\alpha = \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\alpha}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\alpha}{\sin\alpha}$$

$$= \frac{\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{\sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{\cos^2\alpha + \sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{1}{\sin\alpha\cos\alpha} = \frac{1}{\sin\alpha}\cdot\frac{1}{\cos\alpha}$$

$$= \csc\alpha \cdot \sec\alpha$$

Suppose $\sin x = \frac{-2}{3}$

Find

1) $\csc x = \frac{1}{\sin x}$

$$= \frac{1}{-2/3}$$

$$= 1 \div \frac{-2}{3}$$

$$= 1 \cdot \frac{-3}{2} = \boxed{\frac{-3}{2}}$$

2) $\cos x$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{-2}{3}\right)^2 + \cos^2 x = 1$$

$$\frac{4}{9} + \cos^2 x = 1$$

$$\cos^2 x = \boxed{1} - \frac{4}{9}$$

$$\cos^2 x = \frac{9}{9} - \frac{4}{9}$$

$$\cos^2 x = \frac{5}{9}$$

$$\cos x = \pm\sqrt{\frac{5}{9}} = \boxed{\pm\frac{\sqrt{5}}{3}}$$

$\sec\alpha = 3$ find $\tan\alpha$

$1 + \tan^2\alpha = \sec^2\alpha$

$1 + \tan^2\alpha = 3^2$

$\tan^2\alpha = 8$

$\tan\alpha = \pm\sqrt{8}$

$\tan\alpha = \pm\sqrt{4}\sqrt{2}$

$= \boxed{\pm 2\sqrt{2}}$

Class QZ 2

1) Simplify $(2x-5)(4x^2+10x+25)$

$= 8x^3 + 20x^2 + 50x - 20x^2 - 50x - 125$

$= \boxed{8x^3 - 125}$

2) Factor Completely $x^2 - 12x + 36$

$= (x-6)(x-6)$

$= \boxed{(x-6)^2}$